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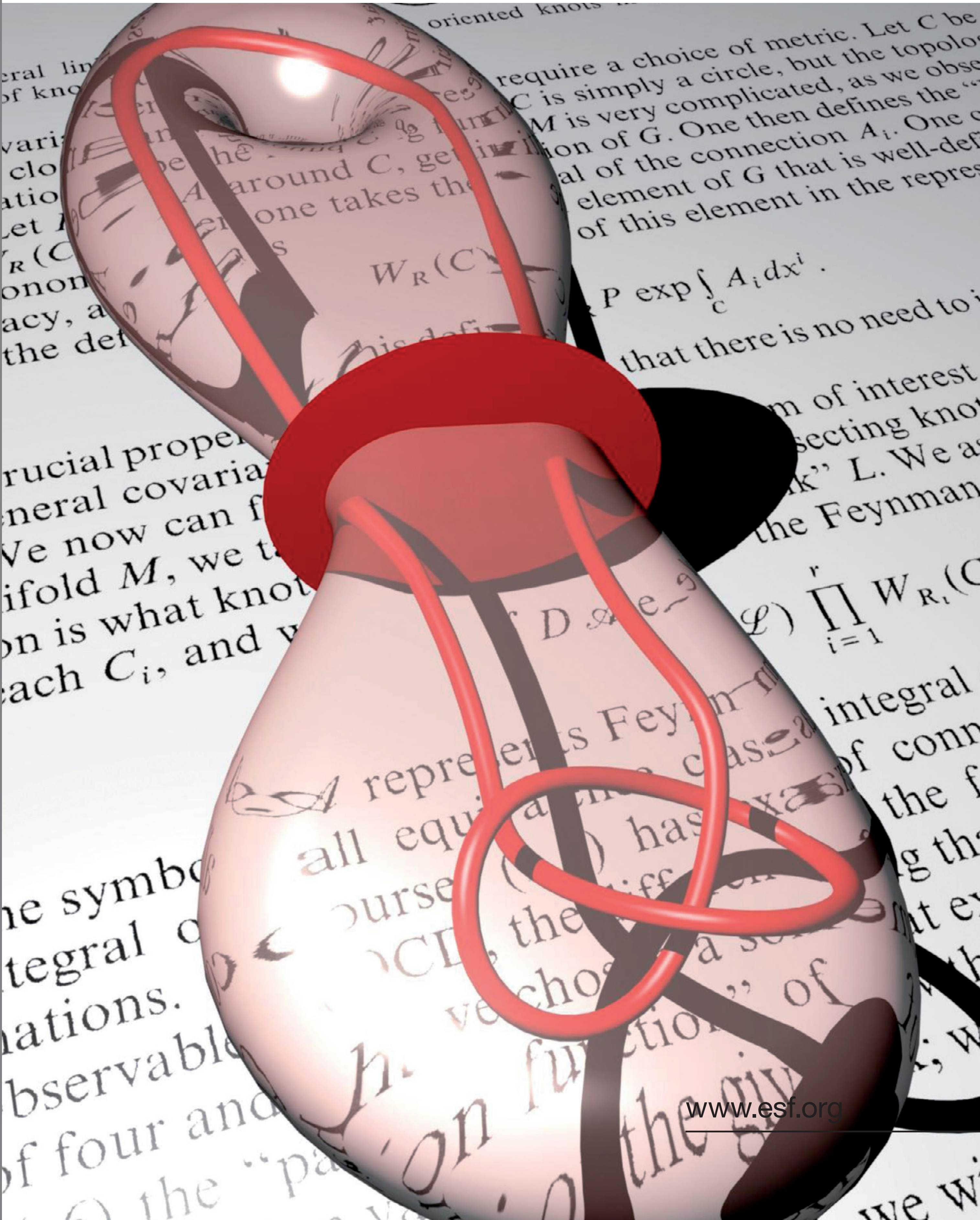
RESEARCH NETWORKING PROGRAMME

INTERACTIONS OF LOW-DIMENSIONAL TOPOLOGY AND GEOMETRY WITH MATHEMATICAL PHYSICS (ITGP)

Standing Committee for Physical and Engineering Sciences (PESC)



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Introduction

The goal of the ESF Interactions of Low-Dimensional Topology and Geometry with Mathematical Physics (ITGP) Research Networking Programme is to facilitate, stimulate and further promote the many interactions of low-dimensional topology and geometry with various fields including gauge theory, quantum topology, symplectic topology and geometry, Teichmüller theory, hyperbolic geometry, string theory and quantum field theory.

The network is organised on a Europe-wide scale, reflecting the global nature of the ongoing research in these areas. The activities – workshops and conferences, schools and programmes of research visits – cross international and disciplinary lines to stimulate current and future progress.

At the same time, this network brings leading experts in the above-mentioned areas together with a new generation of researchers, providing them with the interdisciplinary perspective and training which will plant seeds for the breakthroughs of the future.

The running period of the ESF ITGP Research Networking Programme is for five years, from April 2009 to April 2014.

Scientific Background

Low-dimensional topology and geometry comprise one of the currently most active areas in mathematics, honoured by a number of Fields Medals in the recent past, starting with Thurston and Yau in 1982, Donaldson and Freedman in 1986, Witten and Jones in 1990, McMullen and Kontsevich in 1998 as well as Okounkov and Perelman in 2006. The recent vitality of these fields is largely due to interactions with theoretical physics that have seen dramatic developments over the last three decades.

In the 1970s the interaction between classical Yang-Mills gauge theories and integrable field theories with geometry and topology was pioneered by Atiyah and Faddeev, among others.

In the early 1980s Donaldson's celebrated theorems showed that the h-cobordism theorem fails for 4-manifolds. This failure has many unexpected consequences, most notably the existence of an uncountable number of exotic 4-dimensional Euclidean spaces, when combined with Freedman's proof of the topological Poincaré conjecture in dimension 4. Since then the study of manifolds of dimension less than or equal to 4 (e.g., 3- and 4-dimensional topology, knots in 3-manifolds, mapping class groups of surfaces) has formed a new branch of geometry and topology, called *low-dimensional geometry and topology*.

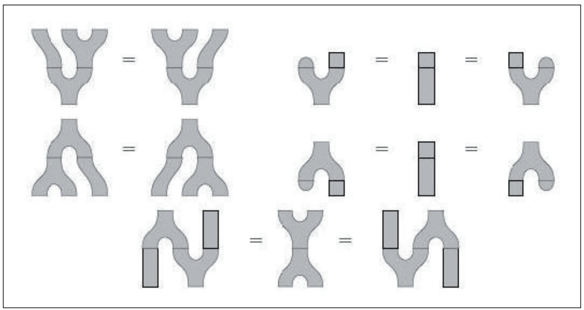
Further interest in low-dimensional topology was provided by Jones' construction of a new and powerful knot invariant arising from his basic constructions for sub-factors, which are also deeply linked to mathematical physics.

The interaction between low dimensional topology and *quantum* field theory was provided by Witten in the late 1980s, when he established that the Jones polynomial is a Wilson loop expectation value in quantum Chern-Simons theory in three dimensions. His observation, that this theory is completely solvable via conformal field theory, led to the rigorous construction of this theory via quantum groups by Reshetikhin and Turaev shortly thereafter.

Since then, the effort to understand classical and quantum field theories as well as string theories, in all their ramifications, has motivated some of the most extraordinary advances in mathematics of the last three decades. This progress has had a profound impact in all the areas of mathematical research included in the present network, some of which have been connected in totally unexpected ways. The most spectacular developments include:

- Kontsevich's universal Vassiliev invariant of links.
- The discovery of Mirror symmetry by Vafa, Greene, Candelas and others, and its relation to T-duality by Strominger, Yau and Zaslow.

- Le, Murakami and Ohtsuki's universal invariant for homology spheres which vastly generalised the Casson invariant.
- Kontsevich's proof of the mirror conjecture.
- Quantum cohomology relating back to Gromov's study of pseudo holomorphic curves in symplectic geometry.
- Kontsevich's proof of Witten's conjecture on the cohomology ring structure of the tautological classes.
- Seiberg-Witten equations and their simplification of Donaldson invariants.
- Taubes' relation between solutions of the Seiberg-Witten equations and pseudo holomorphic curves.
- Donaldson's results on almost holomorphic sections of high powers of prequantum line bundles over symplectic manifolds.
- Seiberg-Witten equations and their simplification of Donaldson invariants, u-plane integrals, generalised Donaldson invariants and its derivation from Supersymmetric Gauge theory.
- Ozsvath-Szabo Floer homology.
- Kashaev's volume conjecture and its relation to the Jones polynomial by Murakami and Murakami.
- Deformation quantisation of all Poisson manifolds by Kontsevich.
- Construction of a canonical deformation quantisation of any compact Kähler manifold via explicit asymptotic analysis of Berezin-Toeplitz operators.
- Symplectic Floer homology of Eliashberg and Hofer.
- The 'AJ'-conjecture of Gukov, further elaborated by Garoufalidis and Le.
- Khovanov's construction of a new link homology, which provides a categorification of polynomial link invariants.
- Kaspustin and Witten's electric-magnetic duality interpretation of the geometric Langlands programme, relation between topological gauge theories, supersymmetric vacua and Bethe ansatz.



Axioms for Frobenius algebras, determining 2-dimensional Topological Quantum Field Theories.

There are three spectacular developments in 3-manifold theory, which combine in order to complete the geometrisation programme initiated by Thurston some three decades ago. Perelman proved the Poincaré conjecture and Thurston's geometrisation conjecture using and greatly extending Hamilton's work in PDEs on Ricci flow. Minsky, Masur, Brock and Canary jointly settled Thurston's ending lamination conjecture. Agol and independently Calegari together with Gabai showed Marden's tame ends conjecture, both using techniques of combinatorial topology and hyperbolic spaces.

Below we discuss in more detail several broad topics this network is pursuing. Some of these topics are already strongly related in many ways, while other relations are under development. A prime objective of the Research Networking Programme is to promote further relations and connections between all of these perspectives.

Topic Areas

Gauge theory

Since the work of Donaldson in the 1980s, gauge theory has evolved into an indispensable tool in the study of smooth 4-manifolds. After the introduction of the Seiberg-Witten invariants in 1994, the theory was both simplified and extended, and the relation to Gromov-Witten invariants (for symplectic 4-manifolds) was established by Taubes. The equivalence of the Donaldson and Seiberg-Witten invariants, conjectured by Witten in 1994, now seems close to being proved by Feehan-Leness. Kronheimer-Mrowka have used part of the work of Feehan-Leness combined with that of Taubes to complete the proof of Property P for knots; later they reproved this more directly using sutured Floer homology. Recently, the Seiberg-Witten invariants together with the rational blow-down technique of Fintushel-Stern have been applied to prove the existence of exotic smooth structures on $CP^2\#k(-CP^2)$ for small values of k according to work of Akhmedov-Park after initial results of J. Park.

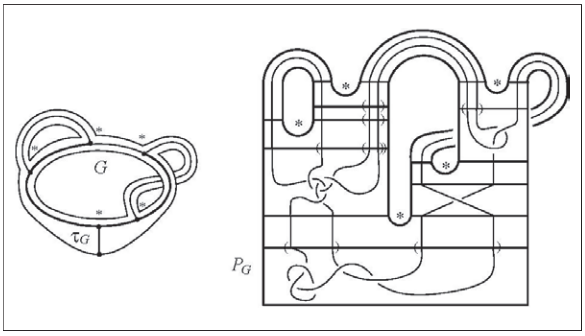
The introduction of 'finite-dimensional approximation' by Furuta signaled a new direction in gauge theory. He first used it to prove a weakened version of the 11/8 conjecture, the 10/8 theorem. Later, Bauer-Furuta defined the *refined* Seiberg-Witten invariants, which are stronger than the classical ones. However, Furuta-Kametani-Matsue-Minami have recently shown that they are not enough to show the 11/8 conjecture.

In the theory of knots and 3-manifolds, the various kinds of Heegard Floer homologies introduced by Ozsvath-Szabo have become a highly active area of research. It is expected that the Heegard Floer homologies for 3-manifolds are essentially isomorphic to the monopole Floer homologies constructed by Kronheimer-Mrowka. Different schemes and ideas towards proving this conjecture have been presented by Y.-J. Lee and Hutchings-Sullivan. There is furthermore major work by Salamon-Wehrheim towards the classical Atiyah-Floer conjecture, which motivated Ozsvath-Szabo's construction.

Three-dimensional hyperbolic geometry

The most dramatic recent development in three-dimensional hyperbolic geometry is the proof of the Poincaré conjecture and the completion of Thurston's geometrisation programme by Perelman. It builds on and greatly extends Hamilton's Ricci flow programme.

Recent solutions of the tameness conjecture by Agol/Calegari-Gabai and the ending lamination conjecture by Minsky-Mazur-Brock-Canary have completed Thurston's programme of classifying hyperbolic structures on a given 3-manifold with finitely generated fundamental group.



A fatgraph G marked in the twice-punctured torus $\Sigma_{1,2}$, the maximal tree τ_G, and a knot in admissible position with respect to the polygonal decomposition P_G.

Agol, Storm and Thurston also apply their results to provide evidence for the positivity of the universal TQFT pairing for 3-manifolds as conjectured by Freedman. Very recently, Freedman, Calegari and Walker have proven that the universal 3-manifold pairing is indeed positive using the new machinery provided by Perelman, Agol, Storm, Thurston and others. We would like to understand the implications of this for TQFT.

The moduli spaces of flat G-connections on 3- and 2-dimensional manifolds can be interpreted as evolution operators and phase spaces of the Chern-Simons field theory, respectively. The Einstein gravity in 3d can be also interpreted as a version of Chern-Simons field theory. In Euclidean space-time and for negative cosmological constant, the roles of the 2d and 3d flat connections are played by the quasi-Fuchsian and the Kleinian groups, respectively. Both spaces are open subsets of the moduli of flat SL(2,C)-connections. The theory of quasi-Fuchsian and Kleinian groups is a rapidly developing subject due to Perelman's geometrisation theorem, the Brock-Canary-Minsky ending lamination theorem, and several others. It is also clear that, due to Hitchin's Higgs field construction, the moduli spaces of SL(2,C)- connections can be interpreted as certain subspaces of the moduli of flat connections with affine group G. On the other hand Kashaev's volume conjecture suggests a very close relation between quantum invariants for compact groups and their hyperbolic analogues. One of the aims of the network is to use these ideas to develop a quantisation scheme for 3d gravity.

Combinatorics of moduli spaces and Quantum Teichmüller theory

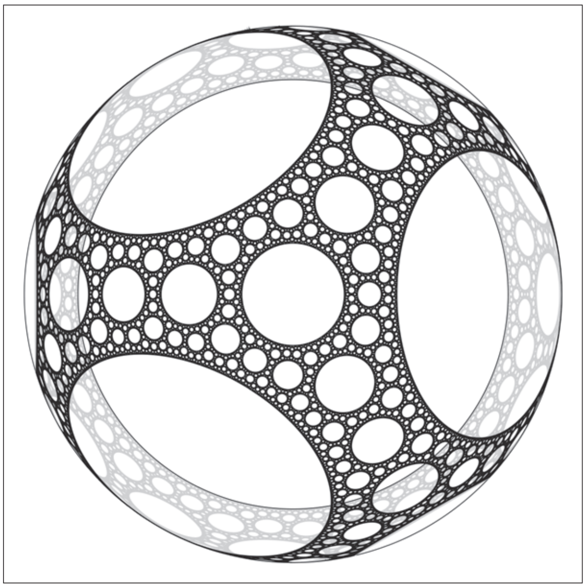
The moduli space of flat G-connections on a smooth punctured surface S is an algebraic manifold admitting several descriptions, using methods relying on different structures on the surface. One of the approaches to understanding these spaces was initiated by Thurston, Strebel, Mumford, Harer and Penner and originally applied to the Teichmüller space [the case where G=PSL(2,R)]. In this approach, the topology of the surface is encoded into combinatorics of its triangulations with vertices in the punctures or, equivalently, of the dual fat graph. This yields a cell decomposition of the moduli spaces of curves by cells numerated by fat graphs, thus reducing the study of topology of the moduli space to combinatorics. This reduction was used by Kontsevich to compute the intersection numbers of the Mumford cycles on moduli space. Closely related are explicit combinatorial coordinates on Teichmüller space, introduced by Penner. These coordinates have been generalised in many ways, namely to moduli spaces of flat F-connections with G a simple Lie group over any ground field, and to the higher analogues of Teichmüller spaces introduced by Hitchin. This combinatorial approach to Teichmüller spaces allowed Kashaev-Chekhov-Fock to find a noncommutative equivariant (quantum) deformation of Teichmüller space. Teschner proved that the representation spaces of these quantum algebras are isomorphic to the spaces of conformal blocks of the Liouville conformal field theory.

These perspectives open the door to manifold possible applications of combinatorial techniques in understanding classical and quantum aspects of Teichmüller space, the mapping class groups, and analogous spaces of representations.

Geometry and Quantisation of Moduli Spaces

Higgs bundles over Riemann surfaces were introduced by Hitchin in 1987 in the study of the self-duality Yang-Mills equations. A Higgs bundle is a pair consisting of a holomorphic bundle and a holomorphic one-form with values in an adjoint bundle. The moduli space of Higgs bundles with a complex reductive structure group has a rich geometric structure. For example, it is a completely integrable system as well as a hyper-Kähler space. Moreover, it can be identified with the moduli space of representations of the fundamental group of a surface in the complex reductive group. The complete integrable system on the moduli space of Higgs bundles has been used in Hitchin's proof of the projectively flatness of his connection in the bundle over Teichmüller spaces which arises by geometric quantisation of the moduli space of semi-stable bundles. More recently, it appears in a prominent way in Witten and Kapustin's electro-magnetic duality interpretation of the geometric Langlands programme.

The consequence of gauge/Bethe correspondence mentioned above turned out to be the description of equivariant intersection theory on the moduli space of Higgs bundles in terms of topological quantum field theory in two-dimensions and Bethe ansatz e.g. work of Gerasimov, Moore, Nekrasov and Shatashvili, as well the quantization of Higgs bundles in terms of equivariant instanton partitions functions.



A Sierpinsky Carpet limit set (McMullen, Curtis; http://www.math.harvard.edu/~ctm/)

Berezin-Toeplitz operators provide natural quantum operators in the context of compact Kähler manifolds. These are asymptotically equivalent to the quantum operators arising in geometric quantisation. It is known that the Berezin-Toeplitz quantisation scheme exhibits an excellent semi-classical behaviour and there is a unique star product associated to it. The relation between Berezin-Toeplitz operators and Hitchin's projectively flat connection has been understood by the work of Andersen. This has led to a proof of Turaev's asymptotic faithfulness conjecture. By further considering coherent state constructions and applying results of Boutet de Monvel, Guillemin and Sjøstrand, Andersen has established that the mapping class groups do not have Kazhdan's property T. Parts of this programme have been achieved so far merely in the non-singular setting. The problem of fully extending it to singular moduli spaces is an active area of research.

Activities

The activities of the ESF ITGP Research Networking Programme will mainly consist of funding science meetings such as workshops, conferences or schools. The ITGP will also be giving grants to researchers within the ITGP research field in the form of short visit grants or exchange grants. Both types of grant are aimed at promoting networking activities between European research institutions.

In particular, funding is available for the following activities:

- Science meetings such as workshops, conferences or schools
- Short research visits of up to 15 days
- Longer exchange visits, from 15 days to three months.

Deadlines for the submission of proposals for science meetings can be found on the website.

There is no deadline for the submission of an application for a short visit or exchange grant. Applications will be reviewed at the end of every month.

In addition to these activities, the ESF ITGP Research Networking Programme wishes to connect the research environment in the ITGP research field through information provided via a website as well as a mailing list for news and listings of events. The goal is to facilitate, stimulate and further promote the interactions of low-dimensional topology and geometry with various fields of research in Europe.

For further information, please see **www.itgp.net** or contact Jane Jamshidi at **jamshidi@qgm.au.dk**.

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