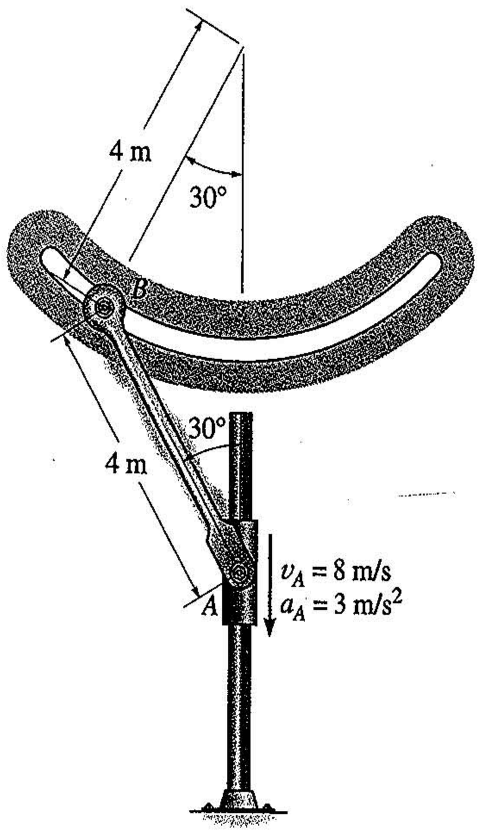
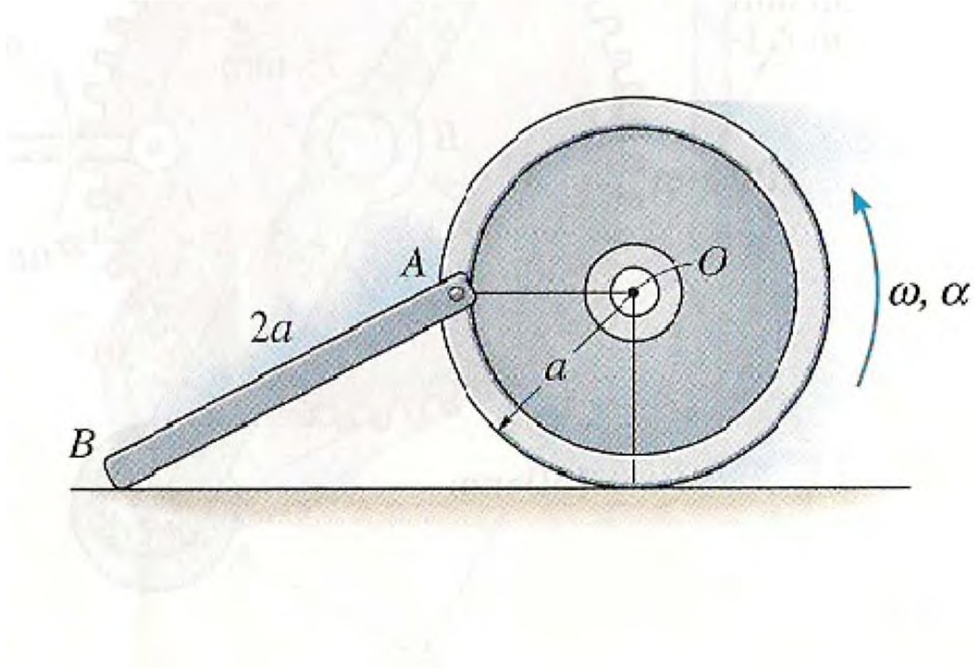
Assignment 1

The ends of the bar AB are confined to move along the paths shown.
At a given instant, A has a velocity of 8 m/s and an acceleration of 3 m/s^2 . Determine the angular velocity and angular acceleration of AB at this instant.



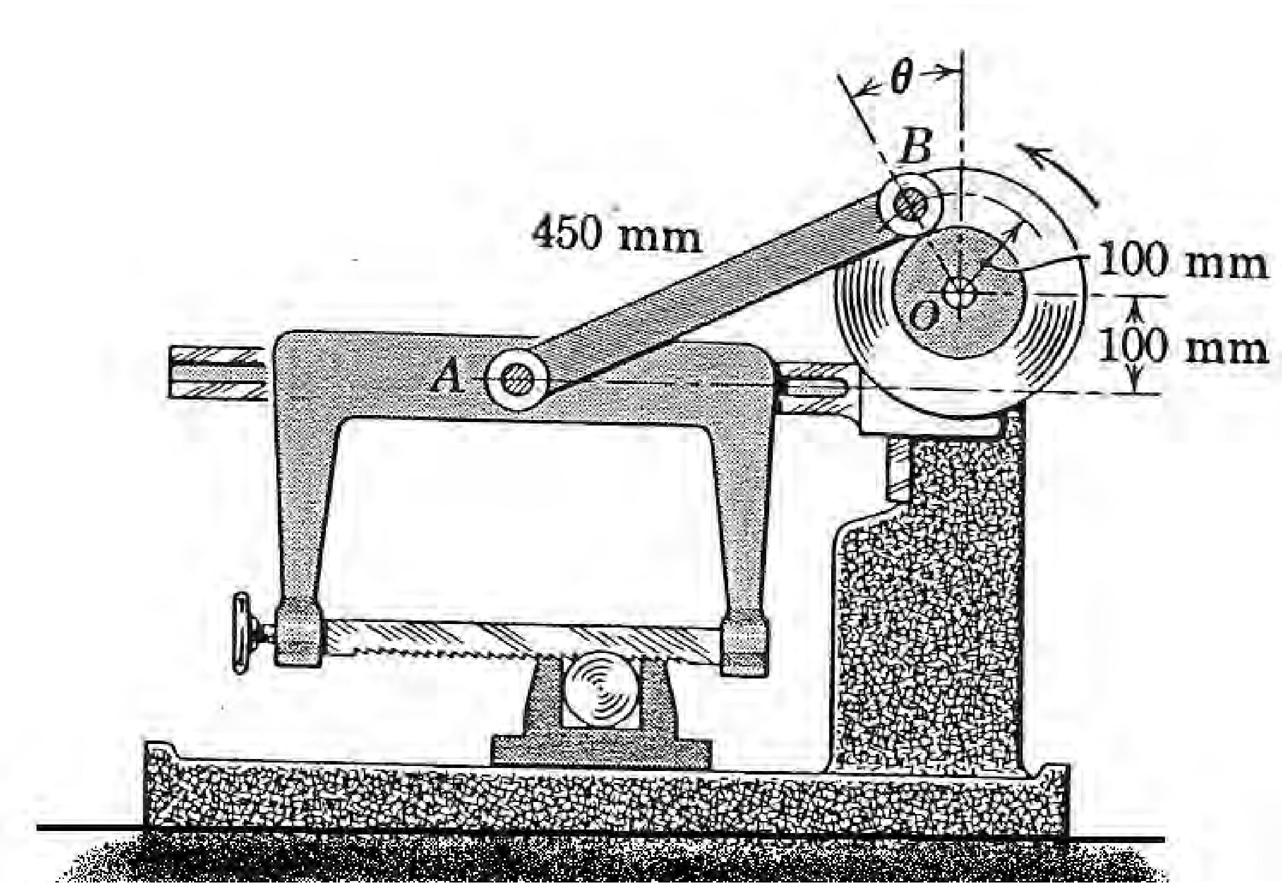
Assignment 2

The wheel rolls without slipping such that at the instant shown it has
An angular velocity ω and angular acceleration α . Determine the
velocity and acceleration of point B on the rod at this instant.

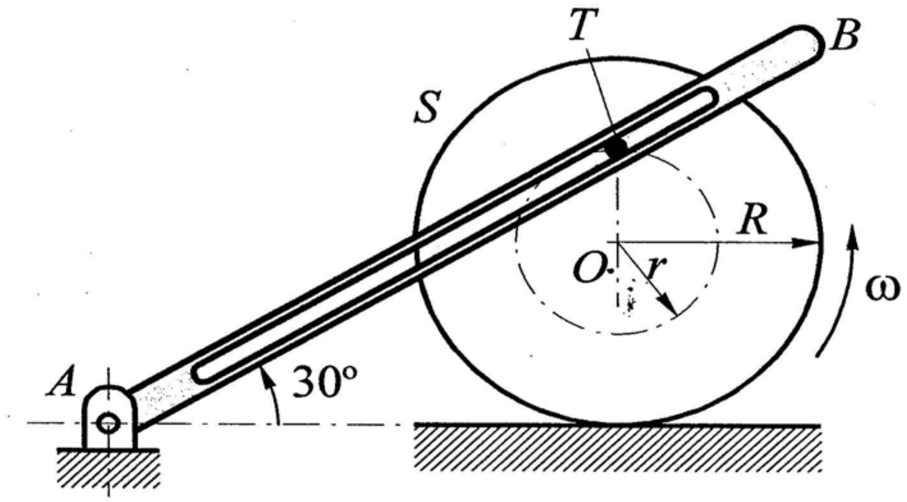


Assignment 3

The elements of a power hacksaw are shown below. The saw blade is mounted in a frame which slides along the horizontal guide. If the motor turns the flywheel at a constant counterclockwise speed of 60 rev/min, determine the acceleration of the blade for the position where $\theta = 90^\circ$, and find the corresponding angular acceleration of the link AB.



Assignment 4



S disc with radius r and center O rolls on the ground with constant angular velocity.
On the disc is T attached in the distance r = R / 2 from the center O.
The rod AB can rotate freely on A and its angle of rotation is controlled by T and the indicated milling in rod AB.

1) Determine the angular velocity of the rod
2) Determine the angular acceleration of the rod

Assignment 5

A thin rigid rod PC with the mass m_{Rod} is hinged friction-free in C to a homogeneous cylinder with the mass m_{Cyl}.

The system is released from rest at the indicated position.
The gravitational acceleration, g has the direction as shown in the figure

Data:

$m_{Rod} = 0.40\,\text{kg}$

$m_{Cyl} = 1.0\,\text{kg}$

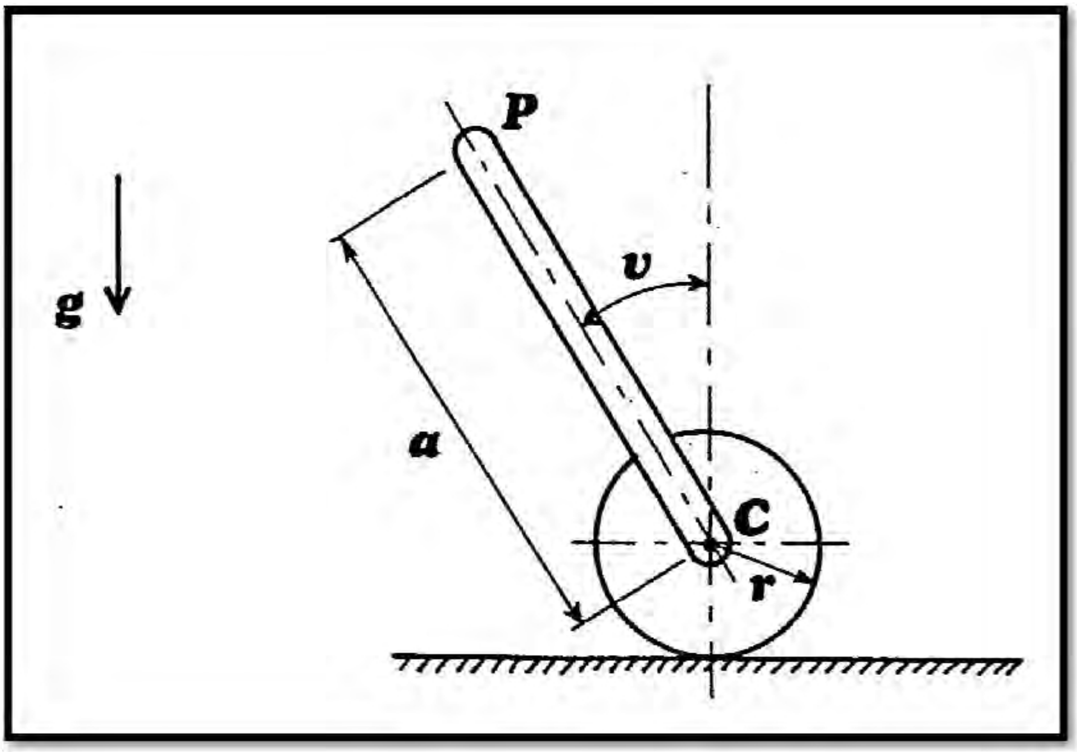
$v = 40^\circ$

$a = 1.0\,\text{m}$

$r = 0.25\,\text{m}$

$g = 9.81\,\text{m/s}^2$

1. Determine the smallest value of the static friction coefficient between the cylinder and the horizontal surface if the cylinder must roll without slipping when it starts

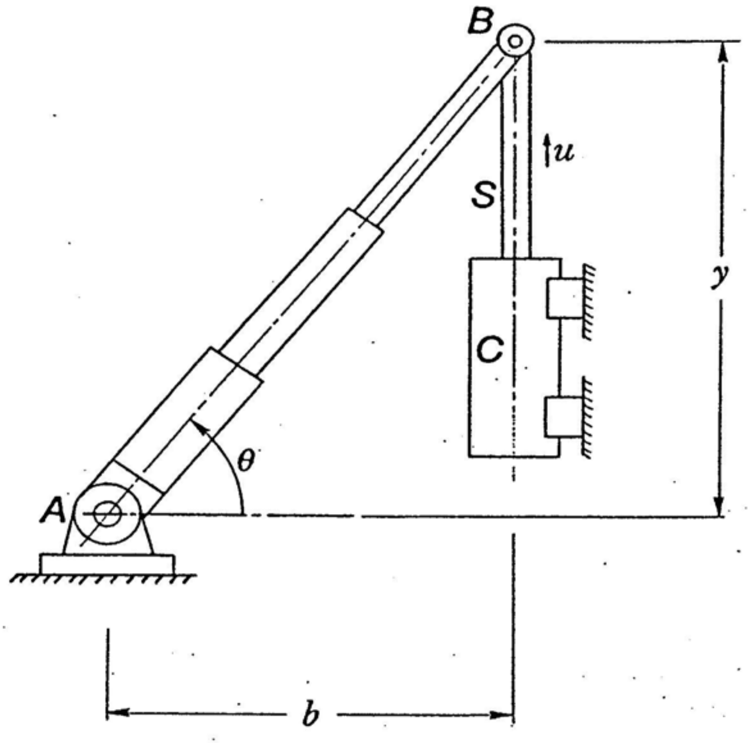


Assignment 6

A Telescope tube AB is hinged at A and in B it is connected to the piston rod S with a rotating joint. The piston rod launched by the fixed pneumatic cylinder C with the constant velocity u = 0.3 [m/s]. For the position shown where b = 0.3 [m] and y = 0.4 [m]the following questions must be answered:

1. Determine the angular velocity $\frac{d\theta}{dt}$ of the Telescope tube AB

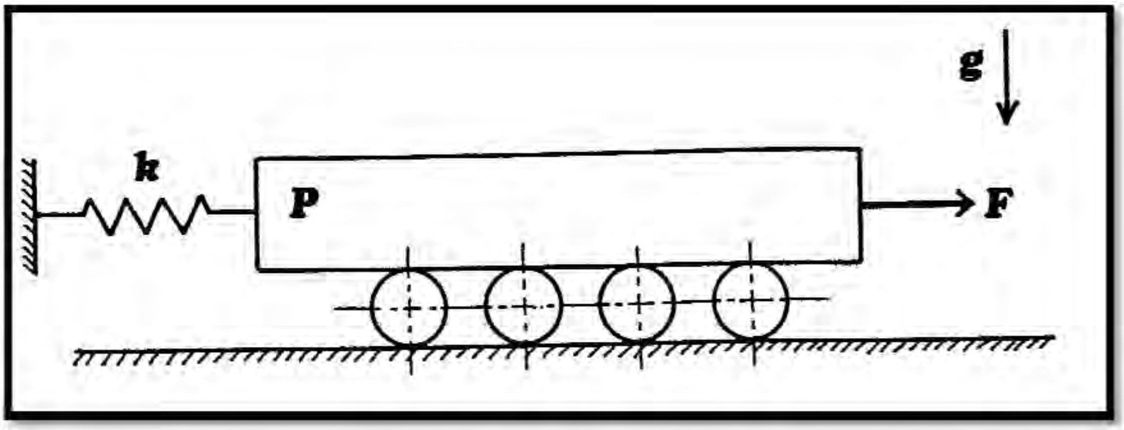
2. Determine the angular acceleration $\frac{d^2\theta}{dt^2}$ of the telescope tube AB



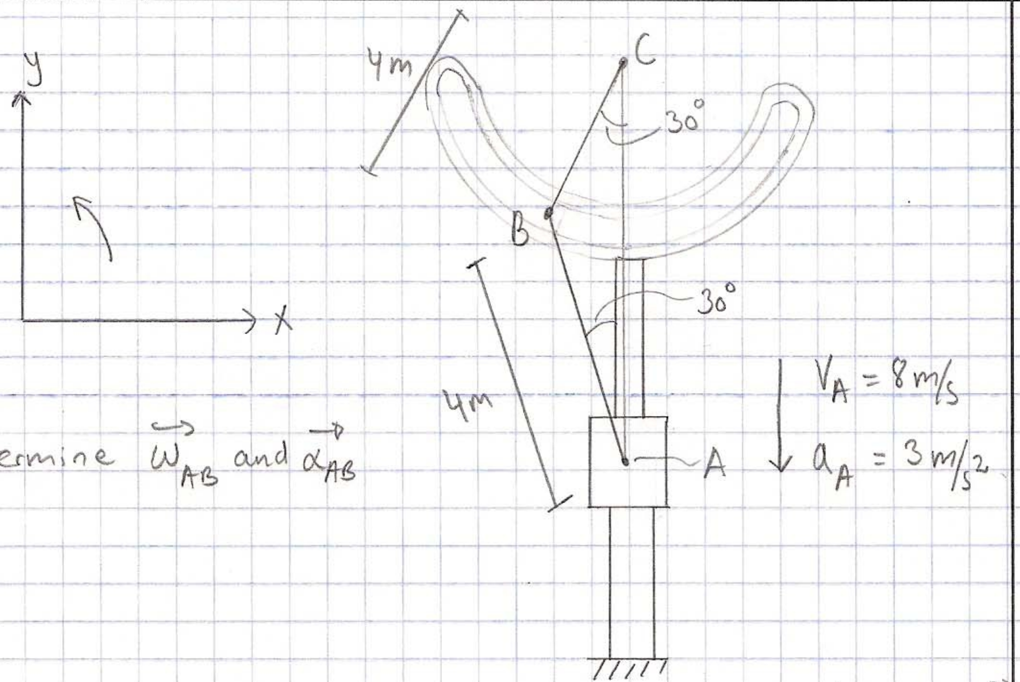
Assignment 7

The figure shows a system consisting of a rectangular homogeneous plate P, which is located on four equally homogeneous cylinders. Plate P has the mass $m_p = 22\,kg$, and each of the cylinders have the mass $m_p = 1.0\,kg$ and a radius $r = 0.010\,m$. At one end of the plate is attached a spring with the spring constant $k = 900\,N/m$. The system is released from rest with the spring undeformed and at the same time the plate is given a constant horizontal force $F = 100\,N$. The gravitational acceleration, g and the direction as shown in the figure. It can be assumed that there is no sliding friction during motion.

1
Determine the speed of the plate when it has achieved a displacement of 200 mm.



1.



Determine $\vec{\omega}_{AB}$ and $\vec{\alpha}_{AB}$

(1)

$\vec{V}_B = \vec{\omega}_{BC} \times \vec{r}_{B/C}$

$\vec{V}_B = \omega_{BC} \vec{k} \times (-2\vec{i} - 3{,}464\vec{j})$

$\Downarrow$

$\vec{V}_B = -2\omega_{BC}\vec{j} + 3{,}464\,\omega_{BC}\vec{i}$

$\vec{r}_{B/A} = -4 \cdot \sin(30°) \cdot \vec{i} + 4\cos(30°)\vec{j}$

$\Downarrow$

$\vec{r}_{B/A} = -2\vec{i} + 3{,}484\vec{j}$

$\vec{r}_{B/C} = -4 \cdot \sin(30°)\vec{i} - 4 \cdot \cos(30°)\vec{j}$

$\Downarrow$

$\vec{r}_{B/C} = -2\vec{i} - 3{,}464\vec{j}$

2. $\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$\vec{V}_B = -8\vec{j} + \omega_{AB} \cdot \vec{k} \times (-2\vec{i} + 3{,}464\vec{j})$

$\Downarrow$

$\vec{V}_B = -8\vec{j} - 2\omega_{AB}\vec{j} - 3{,}464\,\omega_{AB}\vec{i}$

(3)

$-2\omega_{BC}\vec{j} + 3{,}464\,\omega_{BC}\vec{i} = -8\vec{j} - 2\omega_{AB}\vec{j} - 3{,}464\,\omega_{AB}\vec{i}$

$\vec{i}: \quad 3{,}464\,\omega_{BC} = -3{,}464\,\omega_{AB}$

$\vec{j}: \quad -2\omega_{BC} = -8 - 2\omega_{AB}$

$\omega_{AB} = -2\,[r/s] \wedge \omega_{BC} = 2\,[r/s]$

$\Rightarrow$

$\vec{\omega}_{AB} = -2\vec{k}\,[r/s] \wedge \vec{\omega}_{BC} = 2\vec{k}\,[r/s]$

①

$$\vec{a}_B = \vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}^2 \cdot \vec{r}_{B/C}$$

$$\vec{a}_B = \alpha_{BC}\vec{k} \times (-2\vec{i} - 3,464\vec{j}) - 2^2(-2\vec{i} - 3,464\vec{j})$$

$$\Downarrow \quad \vec{a}_B = -2\alpha_{BC}\vec{j} + 3,464\alpha_{BC}\vec{i} + 8\vec{i} + 13,86\vec{j}$$

②

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \cdot \vec{r}_{B/A}$$

$$\vec{a}_B = -3\vec{j} + \alpha_{AB} \cdot \vec{k} \times (-2\vec{i} + 3,464\vec{j}) - (-2)^2 \cdot (-2\vec{i} + 3,464\vec{j})$$

$$\Downarrow \quad \vec{a}_B = -3\vec{j} - 2\alpha_{AB}\vec{j} - 3,464\alpha_{AB}\vec{i} + 8\vec{i} - 13,86\vec{j}$$

③

$$-2\alpha_{BC}\vec{j} + 3,464\alpha_{BC}\vec{i} + 8\vec{i} + 13,86\vec{j} =$$

$$-3\vec{j} - 2\alpha_{AB}\vec{j} - 3,464\alpha_{AB}\vec{i} + 8\vec{i} - 13,86\vec{j}$$

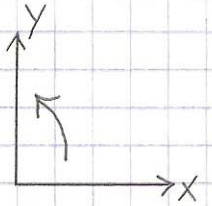
$$\vec{i}: \quad 3,464\alpha_{BC} + 8 = -3,464\alpha_{AB} + 8$$

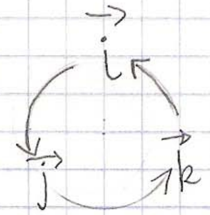
$$\vec{j}: \quad -2\alpha_{BC} + 13,86 = -3 - 2\alpha_{AB} - 13,86$$

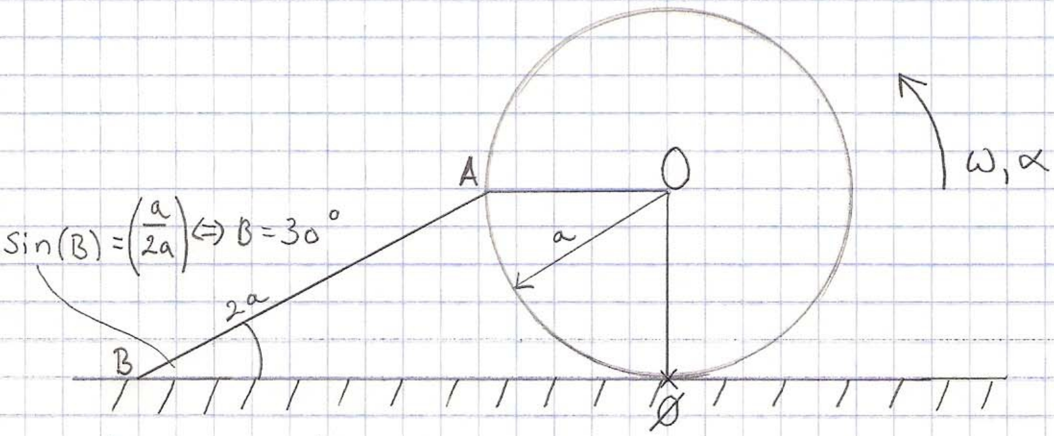
$$\alpha_{AB} = -7,68\,[r/s^2] \;\wedge\; \alpha_{BC} = 7,68\,[r/s^2]$$

$$\Rightarrow \quad \vec{\alpha}_{AB} = -7,68 \cdot \vec{k}\,[r/s^2]$$

Determine : $\vec{V}_B$ and $\vec{a}_B$

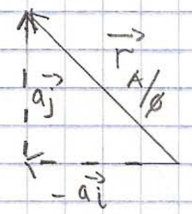






$\vec{r}_{A/Ø} = (-a\vec{i} + a\vec{j})$

$\vec{r}_{A/B} = \sqrt{3}\cdot a\vec{i} - a\vec{j}$



$\vec{r}_{A/0} = -a\cdot\vec{i}$

$\vec{V}_A = \vec{V}_Ø + \vec{\omega} \times \vec{r}_{A/Ø}$

$\vec{V}_A = \omega\vec{k} \times (-a\vec{i} + a\vec{j})$

$\Downarrow$

$\vec{V}_A = -a\omega\vec{j} - a\omega\vec{i}$

$\vec{V}_A = \vec{V}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B}$

$\vec{V}_A = V_B\vec{i} + \omega_{AB}\vec{k} \times \cos(30°)\cdot 2a\cdot\vec{i} + \sin(30°)\cdot 2a\vec{j}$

$-a\omega\vec{j} - a\omega\vec{i} = V_B\vec{i} + \sqrt{3}\,\omega_{AB}\cdot a\vec{j} - \omega_{AB}\cdot a\vec{i}$

$\vec{i}$: $-a\omega = V_B - \omega_{AB}\cdot a$

$\vec{j}$: $-a\omega = \sqrt{3}\cdot\omega_{AB}\cdot a \Leftrightarrow \omega_{AB} = -\frac{\omega}{\sqrt{3}}$

Now insert ω_{AB} in $\vec{i}$.

i : $-a\omega = V_B - \left(-\frac{\omega}{\sqrt{3}}\right)\cdot a$

$\Downarrow$ $V_B = -\frac{\omega}{\sqrt{3}}\cdot a - a\omega$

$\Downarrow$ $V_B = -\frac{1}{\sqrt{3}}\cdot a\omega - 1a\omega$

$\Downarrow$ $V_B = -\frac{\sqrt{3}}{3}a\omega - \frac{3}{3}a\omega$

$\Downarrow$

$V_B = -1{,}577\cdot a\omega\ [m/s] \Rightarrow \underline{\underline{\vec{V}_B = -1{,}577a\omega\vec{i}}}\ [m/s]$

$$\vec{a}_A = \vec{a}_O + \vec{\alpha} \times \vec{r}_{A/O} - \omega^2 \cdot \vec{r}_{A/O}$$

$$\vec{a}_A = -a_O\vec{i} + \alpha\vec{k} \times (-a\vec{i}) - \omega^2(-a\cdot\vec{i})$$

$\Downarrow$

$$\vec{a}_A = a\alpha\cdot\vec{i} - a\alpha\vec{j} + a\omega^2\vec{i}$$

$$\omega_{AB}^2 = \left(-\frac{\omega}{\sqrt{3}}\right)^2 = \frac{\omega^2}{3}$$

$$a_O = \alpha r = \alpha a$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha}_{AB} \times \vec{r}_{A/B} - \omega_{AB}^2 \cdot \vec{r}_{A/B}$$

$$\vec{a}_A = a_B\vec{i} + \alpha_{AB}\cdot\vec{k} \times (\sqrt{3}a\vec{i} + a\vec{j}) - \omega_{AB}^2(\sqrt{3}a\vec{i} + a\vec{j})$$

$$\Downarrow\ \vec{a}_A = a_B\vec{i} + \sqrt{3}a\alpha_{AB}\vec{j} - a\alpha_{AB}\vec{i} - \sqrt{3}a\cdot\omega_{AB}^2\vec{i} - a\omega_{AB}^2\vec{j}$$

$$-a\alpha\vec{i} - a\alpha\vec{j} + a\omega^2\vec{i} = a_B\vec{i} + \sqrt{3}a\alpha_{AB}\vec{j} - a\alpha_{AB}\vec{i} - \sqrt{3}a\omega_{AB}^2\vec{i} - a\omega_{AB}^2\vec{j}$$

$$\vec{j}:\ -a\alpha = \sqrt{3}a\alpha_{AB} - a\omega_{AB}^2$$

$$\vec{i}:\ -a\alpha + a\omega^2 = a_B - a\alpha_{AB} - \sqrt{3}a\omega_{AB}^2$$

$\Downarrow$

$$\vec{j}:\ -a\alpha = \sqrt{3}a\alpha_{AB} + a\cdot\frac{\omega^2}{3} \iff \alpha_{AB} = \frac{-a\cdot\frac{\omega^2}{3} - a\alpha}{\sqrt{3}\cdot a} \iff \alpha_{AB} = \frac{\frac{1}{3}\omega^2 - \alpha}{\sqrt{3}}$$

$$\vec{i}:\ -a\alpha + a\omega^2 = a_B - a\left(\frac{\frac{1}{3}\omega^2 - \alpha}{\sqrt{3}}\right) - \sqrt{3}a\cdot\frac{\omega^2}{3}$$

$$a_B = -a\alpha + a\omega^2 + \frac{\frac{1}{3}a\omega^2 - a\alpha}{\sqrt{3}} + \frac{1}{3}\sqrt{3}a\omega^2$$

$\Downarrow$

$$a_B = -a\cdot\alpha + a\omega^2 + 0{,}1925a\omega^2 - 0{,}5774a\alpha + 0{,}5774a\omega^2$$

$\Downarrow$

$$a_B = -1{,}577a\alpha + 1{,}770a\omega^2\ [m/s^2]$$

$\Downarrow$

$$\underline{\underline{\vec{a}_B = (-1{,}577a\alpha + 1{,}770a\omega^2)\cdot\vec{i}}}\ [m/s^2]$$

$$\vec{a_B} = \vec{a_{t,B}} + \vec{a_{n,B}}$$

$$\Downarrow \quad \vec{a_B} = \vec{\alpha} \times \vec{r_{B/0}} + w^2 \cdot \vec{r_{B/0}}$$

$$\Downarrow \quad \vec{a_B} = -w^2 \cdot \vec{r_{B/0}} \Rightarrow \vec{a_B} = 3{,}9478\vec{i}$$

$$\vec{a_A} = \vec{a_B} + \vec{\alpha_{AB}} \times \vec{r_{A/B}} - \omega_{AB}^2 \cdot \vec{r_{A/B}}$$

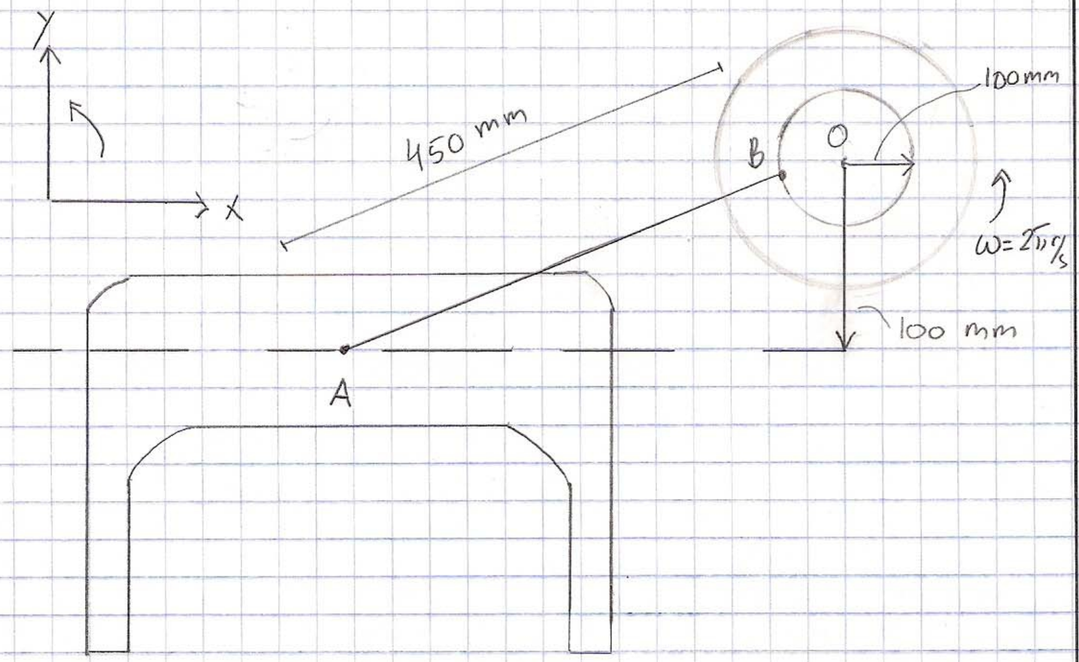
$$\Downarrow \quad a_A \vec{i} = 3{,}9478\vec{i} + \alpha_{AB}\vec{k} \times (-0{,}4387\vec{i} - 0{,}1\vec{j}) - (-1{,}432^2 \cdot (-0{,}4387\vec{i} - 0{,}1\vec{j}))$$

$$\Downarrow \quad a_A \vec{i} = 3{,}9478\vec{i} - 0{,}4387\alpha_{AB}\vec{j} + 0{,}1\alpha_{AB}\vec{i} + 0{,}8996\vec{i} + 0{,}205\vec{j}$$

$$\vec{i}: \quad a_A = 3{,}9487 + 0{,}1\alpha_{AB} + 0{,}8996$$

$$\vec{j}: \quad 0 = -0{,}4387\alpha_{AB} + 0{,}2050$$

$$\Downarrow \quad \vec{a_A} = 4{,}89\vec{i}\ [m/s^2] \quad \wedge \quad \vec{\alpha_{AB}} = 0{,}467\vec{k}\ [r/s^2]$$



$\omega_o =$

$60 \text{ rev} \left(\frac{2\pi \text{ rad}}{60}\right) =$

$6{,}28 \text{ rad/s}$

$\vec{\omega} = 2\pi \vec{k}$ $\alpha = 0$ (because of constant speed)

$\vec{r}_{B/O} = -0{,}1\vec{i}$ $\vec{r}_{A/B} = -0{,}4387\vec{i} - 0{,}1\vec{j}$

$\vec{V}_B = \omega \times \vec{r}_{B/O}$

$\Downarrow$

$\vec{V}_B = 2\pi\vec{k} \times -0{,}1\vec{i} \Rightarrow \vec{V}_B = -0{,}6283\vec{j}$

$\vec{V}_A = \vec{V}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B}$

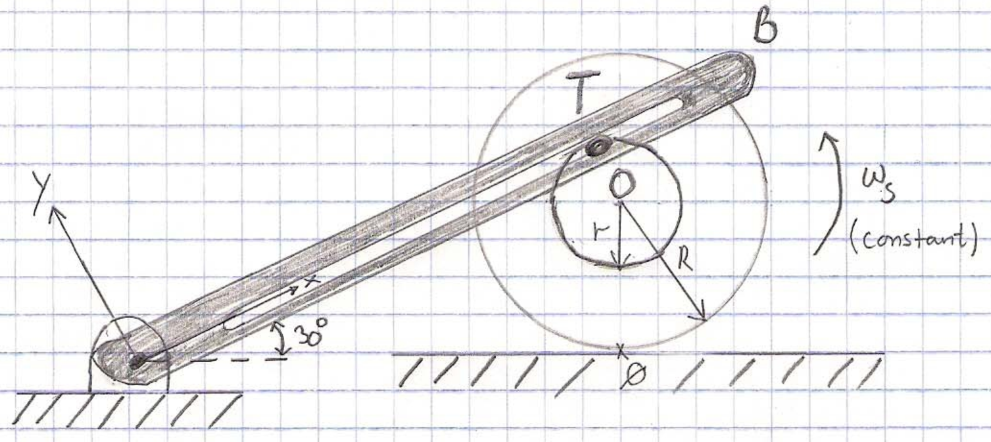
$V_A\vec{i} = -0{,}6283\vec{j} + \omega_{AB}\vec{k} \times (-0{,}4387\vec{i} - 0{,}1\vec{j})$

$\Downarrow$ $V_A\vec{i} = -0{,}6283\vec{j} - 0{,}4387\,\omega_{AB}\vec{j} + 0{,}1\,\omega_{AB}\vec{i}$

$\vec{i}$: $V_A = 0{,}1\,\omega_{AB}$

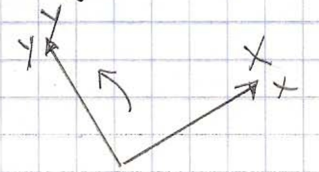
$\vec{j}$: $0 = -0{,}4387\,\omega_{AB} - 0{,}6283$

$\Downarrow$ $\vec{V}_A = -0{,}1432\,\vec{i}$ [m/s] $\wedge$ $\vec{\omega}_{AB} = -1{,}432\,\vec{k}$ [r/s]



$r = \frac{R}{2}$

A rotating reference system with Origo at T attached to AB is used.



Determine $\vec{\omega}_{AB}$ and $\vec{\alpha}_{AB}$

$\vec{V}_T = \vec{V}_\varnothing + \vec{\omega}_S \cdot \vec{r}_{T/\varnothing}$

$\Updownarrow$ $\vec{V}_T = \vec{0} + \omega_s \cdot \vec{k} \times \left(\frac{3 \cdot R}{4} \cdot \vec{i} + \frac{3 \cdot \sqrt{3} \cdot R}{4} \vec{j} \right)$

$\Updownarrow$ $\vec{V}_T = \frac{\omega_s \cdot 3 \cdot R}{4} \vec{j} - \frac{\omega_s \cdot 3 \cdot \sqrt{3} \cdot R}{4} \vec{i}$

$\vec{V}_T = \vec{V}_A + \vec{\Omega}_{AB} \times \vec{r}_{T/A} + \vec{V}_{rel}$

$\Updownarrow$ $\vec{V}_T = \omega_{AB} \cdot \vec{k} \times 3 \cdot R \cdot \vec{i} + V_{rel} \vec{i}$

$\Updownarrow$ $\frac{\omega_s \cdot 3 \cdot R}{4} \cdot \vec{j} - \frac{\omega_s \cdot 3 \cdot \sqrt{3} \cdot R}{4} \cdot \vec{i} = \omega_{AB} \cdot 3R \vec{j} + V_{rel} \vec{i}$

$\vec{i}: -\frac{\omega_s \cdot 3 \cdot \sqrt{3} \cdot R}{4} = V_{rel} \Rightarrow \vec{V}_{rel} = -1,299\, \omega_s \cdot R \vec{i} \left[\frac{m}{s}\right]$

$\vec{j}: \frac{\omega_s \cdot 3 \cdot R}{4} = \omega_{AB} \cdot 3 \cdot R \Rightarrow \vec{\omega}_{AB} = \frac{\omega_s}{4} \cdot \vec{k} \left[\frac{rad}{s}\right]$

$\vec{r}_{T/A} = 3 \cdot R \vec{i}$

$\vec{r}_{T/O} = \frac{R}{4} \cdot \vec{i} + \frac{\sqrt{3} R}{4} \vec{j}$

$\vec{r}_{T/\varnothing} = \frac{3 \cdot R}{4} \vec{i} + \frac{3 \cdot \sqrt{3} \cdot R}{4} \vec{j}$

$\sin(30) = \frac{R + \frac{R}{2}}{hyp}$

$\Updownarrow$

$hyp = 3R$

$\Updownarrow$

$\vec{r}_{T/A} = 3 \cdot R \vec{i}$

$$\vec{a}_T = \vec{a}_0 + \vec{\alpha}_S \times \vec{r}_{T/0} - \omega_S^2 \cdot \vec{r}_{T/0} = -\omega_S^2 \left(\frac{R}{4} \cdot \vec{i} + \frac{\sqrt{3} \cdot R}{4} \cdot \vec{j} \right)$$

$$= -\frac{\omega_S^2 \cdot R}{4} \cdot \vec{i} - \frac{\omega_S^2 \cdot \sqrt{3} \cdot R}{4} \cdot \vec{j}$$

$\dot{\Omega} = \alpha_{AB}$

$\Omega = \omega_{AB}$

$$\vec{a}_T = \vec{a}_A + \vec{\dot{\Omega}}_{AB} \times \vec{r}_{T/A} - \Omega_{AB}^2 \cdot \vec{r}_{T/A} + 2 \cdot \Omega_{AB} \times \vec{V}_{rel} + \vec{a}_{rel}$$

$$\vec{a}_T = \alpha_{AB} \cdot \vec{k} \times 3 \cdot R \cdot \vec{i} - \left(\frac{\omega_S}{4} \right)^2 \cdot 3 \cdot R \cdot \vec{i} + 2 \cdot \frac{\omega_S}{4} \cdot \vec{k} \times$$

$$(-1{,}299) \cdot \omega_S \cdot R \cdot \vec{i} + a_{rel} \vec{i}$$

$$\vec{a}_T = \alpha_B \cdot 3 \cdot R \cdot \vec{j} - \frac{3 \cdot \omega_S^2 \cdot R}{16} \vec{i} - 0{,}65 \cdot \omega_S^2 \cdot R \cdot \vec{j} + a_{rel} \cdot \vec{i}$$

$$\vec{i}: \quad \frac{\omega_S^2 \cdot R}{4} = -\frac{3 \cdot \omega_S^2 \cdot R}{16} + a_{rel}$$

$$\vec{j}: \quad -\frac{\omega_S^2 \cdot \sqrt{3} \cdot R}{4} = \alpha_{AB} \cdot 3 \cdot R - 0{,}65 \cdot \omega_S^2 \cdot R$$

$$\Rightarrow \quad \vec{\alpha}_{AB} = \underline{\underline{0{,}0722 \cdot \omega_S^2 \cdot \vec{k}}} \; [\mathrm{rad}/s^2]$$

$$\vec{a}_{rel} = -0{,}0625 \cdot \omega_S^2 \cdot R \; [m/s^2]$$

Cyl:

(6) $\Sigma F_x = m_{cyl} \cdot \bar{a}_c \iff C_x - F_f = m_{cyl} \cdot \bar{a}_c$

(7) $\Sigma F_y = m_{cyl} \cdot \bar{a}_{c,y} \iff -C_y - m_{cyl} \cdot g + N = 0$

(8) $\curvearrowleft + \Sigma M_G = I_G \cdot \alpha_{cyl} \iff -F_f \cdot r = \frac{1}{2} \cdot m_{cyl} \cdot r^2 \cdot \alpha_{cyl}$

(9) $\bar{a}_c = -\alpha_{cyl} \cdot r$

The equation system is solved in Mathcad. Please see attached Mathcad file.

$\Rightarrow F_f \leq \mu_s \cdot N \qquad F_f = 0{,}42 \qquad N = 12{,}394$

$0{,}42 \leq \mu_s \cdot 12{,}394 \iff \underline{\underline{\mu_s \geq 0{,}034}}$

5

Data :

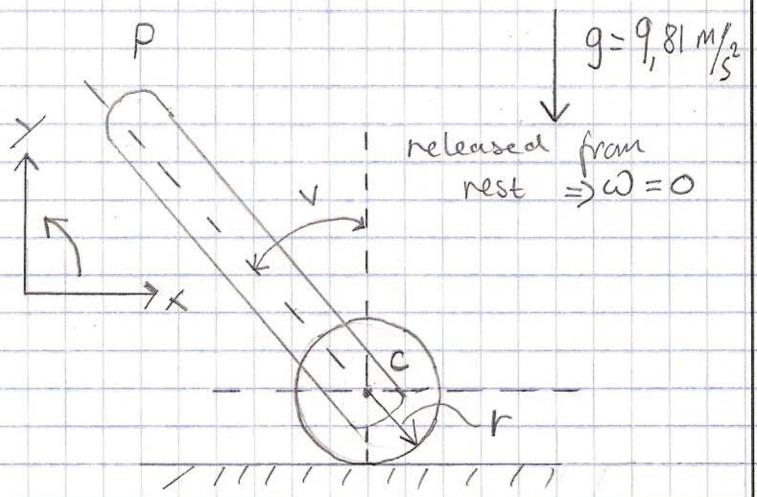
$m_{rod} = 0{,}40 \text{ kg}$

$m_{cyl} = 1{,}0 \text{ kg}$

$v = 40°$

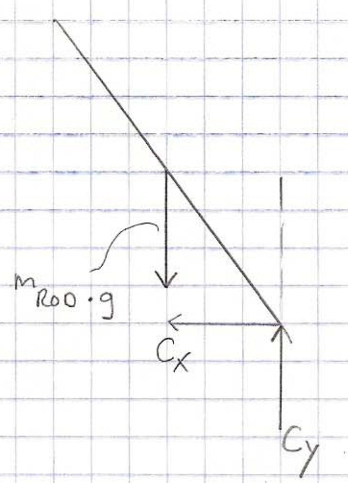
$a = 1{,}0 \text{ m}$

$r = 0{,}25 \text{ m}$



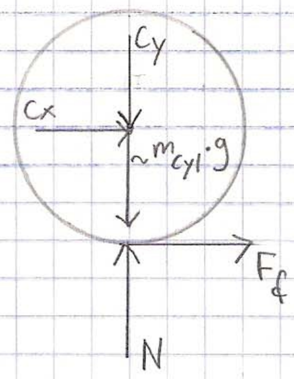
General motion.

FBD ROD



General motion - roll without slipping.

FBD CYL.



ROD :

① $\Sigma F_x = m_{ROD} \cdot \bar{a}_{ROD,x} \iff -C_x = m_{ROD} \cdot \bar{a}_{ROD,x}$

② $\Sigma F_y = m_{ROD} \cdot \bar{a}_{ROD,y} \iff C_y - m_{ROD} \cdot g = m_{ROD} \cdot \bar{a}_{ROD,y}$

③ $\circlearrowleft \Sigma M_G = I_G \cdot \alpha_{ROD} \iff -C_x \cdot \frac{a}{2} \cdot \cos(v) + C_y \cdot \frac{a}{2} \cdot \sin(v) = \frac{1}{12} \cdot m_{ROD} \cdot a^2 \cdot \alpha_{ROD}$

$$\vec{a}_C = \vec{a}_G + \vec{\alpha}_{ROD} \times \vec{r}_{C/G} - \cancel{\omega^2_{ROD} \cdot \vec{r}_{C/G}}$$

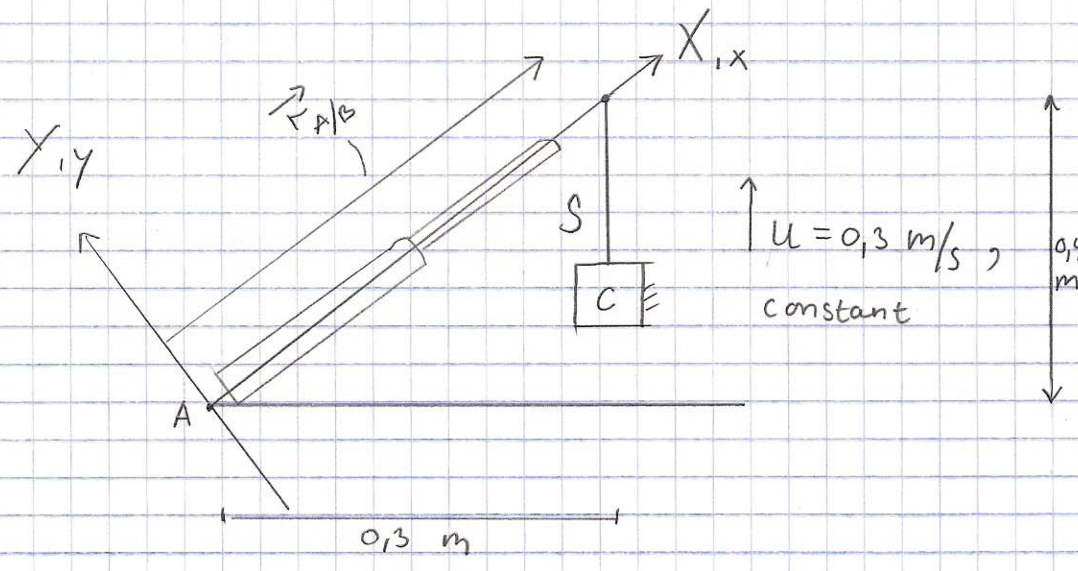
$\Downarrow$ $\bar{a}_C \cdot \vec{i} = \bar{a}_{ROD,x} \cdot \vec{i} + \bar{a}_{ROD,y} \cdot \vec{j} + \alpha_{ROD} \cdot \vec{k} \times \left(\frac{a}{2} \cdot \sin(v) \cdot \vec{i} - \frac{a}{2} \cdot \cos(v) \cdot \vec{j}\right)$

$\Downarrow$ $\bar{a}_C \cdot \vec{i} = \bar{a}_{ROD,x} \cdot \vec{i} + \bar{a}_{ROD,y} \cdot \vec{j} + \frac{a}{2} \cdot \sin(v) \cdot \alpha_{ROD} \vec{j} + \frac{a}{2} \cdot \cos(v) \cdot \alpha_{ROD} \cdot \vec{i}$

④ $\vec{i} : \bar{a}_C = \bar{a}_{ROD,x} + \frac{a}{2} \cdot \cos(v) \cdot \alpha_{ROD}$

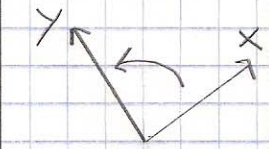
⑤ $\vec{j} : 0 = \bar{a}_{ROD,y} + \frac{a}{2} \cdot \sin(v) \cdot \alpha_{ROD}$

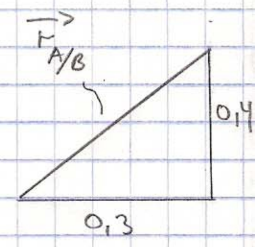
6



A reference system (rotating) with Origo in A attached to the cylinder AB is used.

Reference





$0{,}3^2 + 0{,}4^2 = x^2$

$\Rightarrow x = 0{,}5$

$\vec{\Omega} = \omega_{AB}\vec{k}$

$\vec{r}_{B/A} = 0{,}5\,\vec{i}$

a) Determine ω_{AB}

$\vec{V}_B = \vec{V}_A + \vec{\Omega} \times \vec{r}_{B/A} + \vec{V}_{rel}$

$\Downarrow\ \vec{V}_B = \vec{0} + \omega_{AB}\vec{k} \times 0{,}5\,\vec{i} + V_{rel}\,\vec{i}$

$\Updownarrow\ \vec{V}_B = 0{,}5\,\omega_{AB}\vec{j} + V_{rel}\,\vec{i}$

$\vec{V}_B = 0{,}3 \cdot \frac{0{,}4}{0{,}5}\,\vec{i} + 0{,}3 \cdot \frac{0{,}3}{0{,}5}\,\vec{j}$

$= 0{,}24\,\vec{i} + 0{,}18\,\vec{j}$

$\vec{V}_B = \vec{V}_B$

$0{,}24\,\vec{i} + 0{,}18\,\vec{j} = 0{,}5\,\omega_{AB}\vec{j} + V_{rel}\,\vec{i}$

$\vec{i}:\quad 0{,}24 = V_{rel} \Leftrightarrow V_{rel} = 0{,}24 \Rightarrow \vec{V}_{rel} = 0{,}24\,\vec{i}$

$\vec{j}:\quad 0{,}18 = 0{,}5\,\omega_{AB} \Leftrightarrow \omega_{AB} = 0{,}36 \Rightarrow \vec{\omega}_{AB} = 0{,}36\,\vec{k}$

[r/s]

6

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} + \vec{r}_{B/A} - \omega_{AB}^2 \cdot \vec{r}_{B/A} + 2\vec{\omega}_{AB} \times \vec{V}_{rel} + \vec{a}_{rel}$$

$$= \vec{0} + \alpha_{AB}\vec{k} \times (0{,}5\vec{i}) - 0{,}36^2(0{,}5\vec{i}) + 2 \cdot 0{,}36\vec{k} \times 0{,}24\vec{i} + a_{rel}\vec{i}$$

$$= 0{,}5\alpha_{AB}\vec{j} - 0{,}0648\vec{i} + 0{,}1728\vec{j} + a_{rel}\vec{i}$$

$\vec{a}_B = \vec{0}$, because u is constant

$$\vec{a}_B = \vec{a}_B$$

$$0 = 0{,}5\alpha_{AB}\vec{j} - 0{,}0648\vec{i} + 0{,}1728\vec{j} + a_{rel}\vec{i}$$

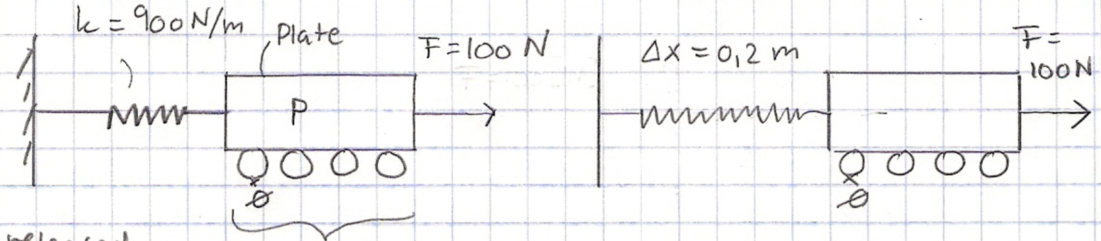
$\vec{i}$: $0 = -0{,}0648 + a_{rel}$

$\vec{j}$: $0 = 0{,}5\alpha_{AB} + 0{,}1728$

$\Leftrightarrow \quad \alpha_{AB} = -0{,}3456 \, [r/s^2]$

$\Rightarrow \quad \vec{\alpha}_{AB} = -0{,}3456\vec{k} \, [r/s^2]$

$m_p = 22\ kg$ $\quad r_{cyl} = 0{,}01\ m$ $\quad m_{cyl} = 1\ kg$ $\quad \downarrow g$



Is released from rest

$\Delta x_1 = 0 \quad V_1 = 0$

Determine the speed of the plate when $\Delta x = 0{,}2\ m$:

$\Delta x = \Delta s$

P = Tranlational Motion

Cyl = General Motion

$W_{nc} = \Delta E_{KIN} + \Delta E_{pot,g} + \Delta E_{pot,e}$

W_{nc} = Work NON CONSERVATIVE

$\Updownarrow$

$F \cdot \Delta s = E_{kin,2P} - E_{kin,1,P} + 4 \cdot E_{kin,2,cyl} - 4 \cdot E_{kin,1,cyl} +$

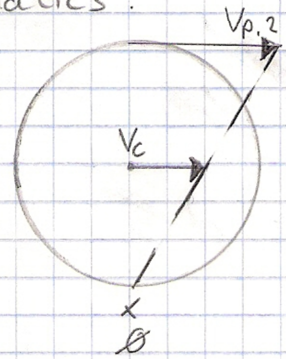
$E_{pot,e,2} - E_{pot,e,1}$

$\Updownarrow$

$F \cdot \Delta s = \frac{1}{2} \cdot m_p \cdot V_{2,P}^2 + 4 \cdot \frac{1}{2} \cdot I_{\emptyset,cyl} \cdot W_{cyl}^2 + \frac{1}{2} k \Delta x^2$

$I_\emptyset = \frac{3}{2} \cdot m_c \cdot r^2$

Kinematics:



$V_{2,P} = 2 \cdot r \cdot \omega_{cyl} \Leftrightarrow W_{cyl} = \frac{V_P}{2r}$

$F \cdot \Delta s = \frac{1}{2} m_p \cdot V_{2,P}^2 + 4 \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot m_{cyl} \cdot r^2 \left(\frac{V_{2,P}}{2 \cdot r}\right)^2 + \frac{1}{2} \cdot k \cdot \Delta x^2$

$\Updownarrow$

$100 \cdot 0{,}2 = \frac{1}{2} \cdot 22 \cdot V_{2,P}^2 + 4 \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot 1 \cdot 0{,}01^2 \cdot \left(\frac{V_{2,P}}{2 \cdot 0{,}01}\right)^2 + \frac{1}{2} \cdot 900 \cdot 0{,}2^2$

$\Rightarrow V_{2,P} = 0{,}4126\ [m/s]$

My Fundamental Equations Of Dynamics

By Marianne Willert

First some rules to remember

- Everything that rolls make a general motion
- A wheel that rolls does not perform any work
- Conservative forces are forces which are not dependent on the road, gravity and spring forces
- NON-conservative forces are frictional force + external forces
- Working for a non - sliding friction force is always 0
- Forces influencing the body in 90° relative to the direction of motion provides no work for example the normal force
- Always Calculate variable as positive - if the result is negative, it is because it really goes the opposite direction
- Know Pythagoras, and sines
- $\Delta =$ End - Begin (end position - start position)

1 Friction

The sphere / cylinder / disc rolls

$$a_G = \alpha \cdot r$$

$$F \le \mu_s \cdot N$$

The sphere / cylinder / disc is slipping (DO NOT USE $a_G = \alpha \cdot r$).

$$F > \mu_s \cdot N$$

$$F = \mu_K \cdot N$$

2 Work and energy

(The equation of work and energy is used to solve problems involving force, velocity, and displacement. Before applying the equation always draw a free-body diagram of the body in order to identify the forces which do work)

Use energy considerations, when given two situations, a start and an end or if the speed is desired (v or ω)

When acceleration is not included, it is smart to use the working equation.

- Energy is always positive
- Check first if there are external influences – is it a conservative system with friction force and gravity or
  is it a NON-conservative system with force, torque and spring force
- Conservative forces are forces caused by external influences such as engines, fluids, etc.If the forces acting on the body are conservative forces, then apply the conservation of energy equation:
  $T_1 + V_1 = T_2 + V_2$
- NON - conservative forces are forces whose job depends on the distance traveled
- Are there differences in levels – remember to show an elevation axis which is the opposite direction to gravity
  Provide working equation

$$A_{ik} = \Delta T + \Delta V_g + \Delta V_e$$

2.1 Translation

$$T = \frac{1}{2} \cdot m \cdot v^2$$

$E_{pot,g} = V_g = m \cdot g \cdot h$, also by general motion and rotation about a fixed axis

$E_{pot,e} = V_e = \frac{1}{2} k \cdot x^2$, also by general motion and rotation about a fixed axis

2.2 Rotation about a fixed axis

$$T = \frac{1}{2} \cdot I_o \cdot \omega^2$$

2.3 General motion

$$T = \frac{1}{2} \cdot m \cdot v^2 + \frac{1}{2} \cdot \bar{I} \cdot \omega^2, \ (\omega^2 \cdot r^2 = v^2)$$

$$T = \frac{1}{2} \cdot I_ø \cdot \omega^2$$

2.4 Forces work

$A = F \cdot s = \vec{F} \cdot \vec{s} = F \cdot s \cdot \cos(\theta)$, hvor θ er vinklen mellem F og s.

3 Planar kinematics of a rigid body

(Kinematics; studying the geometry of motion without any concern for the forces which cause the motion. Before solving a planar kinematics problem you have to classify the motion as being either rectilinear or curvilinear translation, rotation about a fixed axis or a general plane motion.

- Use instantaneous center of zero velocity(IC),ø ONLY if it rools without slipping, than the point of contact with the ground has zero velocity
- First check on the gravitational acceleration, g is included - if not it is a kinematic assignment
- specify the Coordinate System
- If the mechanism is constructed so that there may be slippage of the connected points, establish a rotating reference system. REMEMBER to describe the origo of the reference and its attachment
- Draw the system
- Apply the velocity equation
- Perhaps also the acceleration equation
- Compare coefficients i and j

3.1 General motion

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \overrightarrow{\omega^2} \cdot \vec{r}_{B/A}$$

$$\overrightarrow{V_p} = \vec{\omega} \times \vec{r}_{P/Ø}$$

3.2 Translation

(when the body moves with rectilinear or curvilinear translation, all the points on the body have the same motion)

$$\vec{v}_B = \vec{v}_A$$

$$\vec{a}_B = \vec{a}_A$$

4 Planar kinematics of a rigid body (force and rotation)

- Draw the system
- Specify the coordinate system
- Draw FBD (Free-body diagram) and KD (Kinematic Diagram)
- Newton's laws
- Compare equations and unknowns
- If more unknowns than equations -KD for each body has to be done
- Solve the equations

4.1 Rotation about a fixed axis

$$\overrightarrow{V_p} = \vec{\omega} \times \vec{r}_{P/O} \qquad \left|\vec{v}_P\right| = \left|\vec{\omega}\right| \cdot \left|\vec{r}_P\right| \cdot \sin(\theta)$$

$$\vec{a}_P = \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \cdot \vec{r}_{P/O}$$

$$a_t = \alpha \cdot r \qquad \overrightarrow{a_t} = \overrightarrow{\alpha} \times \overrightarrow{r} \qquad v = \omega \cdot r$$

$$a_n = \omega^2 \cdot r \qquad \overrightarrow{a}_n = -\omega^2 \cdot \overrightarrow{r}$$

$$a_P = \sqrt{a_t^{\,2} + a_n^{\,2}} \qquad \overrightarrow{a}_P = \overrightarrow{a_t} + \overrightarrow{a_n}$$

4.2 Rotating reference system

$$\overrightarrow{v}_B = \overrightarrow{v}_A + \overrightarrow{\Omega} \times \overrightarrow{r}_{B/A} + \overrightarrow{v}_{rel}$$

$$\overrightarrow{a}_B = \overrightarrow{a}_A + \overrightarrow{\dot{\Omega}} \times \overrightarrow{r}_{B/A} - \Omega^2 \cdot \overrightarrow{r}_{B/A} + 2\overrightarrow{\Omega} \times \overrightarrow{v}_{rel} + \overrightarrow{a}_{rel}$$

5 Kinetic (Newton)

(Kinetics i s the study of the relationship between forces and the acceleration they cause. This relationship is based on Newton's second law of motion)

5.1 *Rectilinear translation*

x-y coordinate system

$$\sum \overrightarrow{F} = m \cdot \overrightarrow{\overline{a}}_c$$

$$\sum F = m \cdot \overline{a}_x$$

$$\sum F = m \cdot \overline{a}_y$$

$$\sum \overrightarrow{F} = m \cdot \overrightarrow{\overline{a}}_c$$

$$\sum M_G = 0$$

$$\sum M_P = \sum (M_K)_P$$

5.2 Curvilinear translation

n-t coordinate system

$$\sum F_n = m \cdot \bar{a}_n$$

$$\sum F_t = m \cdot \bar{a}_t$$

$$\sum M_G = 0$$

$$a_n = \frac{v^2}{\rho}$$

5.3 Rotation about a fixed axis

n-t Coordinate system

$$\sum F_n = m \cdot \bar{a}_n = m \cdot \omega^2 \cdot r_G$$

$$\sum F_t = m \cdot \bar{a}_t = m \cdot \alpha \cdot r_G$$

$$\sum M_G = I_G \cdot \alpha$$

$$\sum M_O = I_O \cdot \alpha$$

5.4 General motion

x-y Coordinate system

$$\sum F_x = m \cdot \bar{a}_x$$

$$\sum F_y = m \cdot \bar{a}_y$$

$$\sum M_G = I_G \cdot \alpha \qquad \sum M_P = \sum (M_K)_P$$

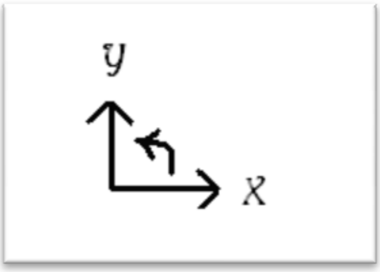
5.5 Parallel-Axis Theorem (Steiner)

$$I = I_G + md^2$$

d = perpendicular distance between the two axes which can be moved from the center of gravity ONLY.

My final advice

Remember to show which direction you caculate!



Have fun☺!

Min Formelsamling til Dynamik på maskiningeniøruddannelsen

Udarbejdet af Marianne Willert

Først nogle huskeregler

- Alt hvad der ruller foretager en general bevægelse
- Et hjul der ruller udfører ikke noget arbejde
- Konservative kræfter er kræfter der ikke er afhængige af vejstrækning, tyngdekraften og fjederkræfter
- IKKE- konservative kræfter er friktionskraften + udefrakommende påførte kræfter
- Hvis det ikke vides, om der er rullende eller glidende friktion, sættes friktionskraften som ubekendt og der regnes som rullefriktion
- Arbejde for en IKKE – glidende friktionskraft er altid 0
- Kræfter der påvirker legemet i 90° i forhold til bevægelsesretningen yder intet arbejde eksempelvis normalkræften
- Sæt aldrig fortegn på variable. Regn dem i stedet ALTID som positive, hvis resultatet så er negativt, er det fordi, at det i virkeligheden går modsat
- Kend pythagoras, $\sin(\theta), \cos(\theta), \tan(\theta)$, samt sinusrelationerne
- $\Delta =$ Slut – Begynd (slutsituation – startsituation)

1 Friktion

Kuglen/cylinderen/skiven ruller

$$a_G = \alpha \cdot r$$

$$F \le \mu_s \cdot N$$

Kuglen/cylinderen/skiven glider ($a_G = \alpha \cdot r$ må IKKE anvendes).

$$F > \mu_s \cdot N$$

$$F = \mu_K \cdot N$$

2 Arbejde og energi

Der regnes ud fra energibetragtninger, når der er angivet to situationer, en start og en slut eller hvis hastigheder ønskes (v eller ω)

- Når acceleration ikke indgår, er det smart at bruge arbejdssætningen
- Arbejde regnes med fortegn
- Energi er altid positivt
- Undersøg først om der er ydre påvirkninger - er det et konservativt system med friktionskraft og tyngdekraft eller
  er det et IKKE- konservativt system med kraft, moment og fjederkraft
- Konservative kræfter opstår ved ydre påvirkninger såsom motorer, væsker m.m
- IKKE – konservative kræfter er kræfter hvis arbejde afhænger af den tilbagelagte strækning
- Er der niveauforskelle - husk derved at indlægge en højdeakse, som er modsatrettet tyngdekraften
- Opstil arbejdsligning

$$A_{ik} = \Delta T + \Delta V_g + \Delta V_e$$

2.1 Translation

$$T = \frac{1}{2} \cdot m \cdot v^2$$

$E_{pot,g} = V_g = m \cdot g \cdot h$, også ved general bevægelse og rotation om fast akse

$E_{pot,e} = V_e = \frac{1}{2} k \cdot x^2$, også ved general bevægelse og rotation om fast akse

2.2 Rotation om fast akse

$$T = \frac{1}{2} \cdot I_o \cdot \omega^2$$

2.3 General bevægelse

$$T = \frac{1}{2} \cdot m \cdot v^2 + \frac{1}{2} \cdot \bar{I} \cdot \omega^2, \ (\omega^2 \cdot r^2 = v^2)$$

$$T = \frac{1}{2} \cdot I_ø \cdot \omega^2$$

2.4 Kræfters arbejde

$A = F \cdot s = \vec{F} \cdot \vec{s} = F \cdot s \cdot \cos(\theta)$, hvor θ er vinklen mellem F og s.

3 Plan kinematik for et stift legeme

- Kontroler først om tyngdeaccelerationen, g indgår – hvis ikke er det en kinematikopgave
- Øjeblikkeligt omdrejningspunkt, ø anvendes kun ved hastighedsbetragtning
- Fastlæg koordinatsystem
- Hvis mekanismen er opbygget således, at der kan opstå glidning i de forbundne punkter, fastlægges et roterende referencesystem. HUSK at beskrive referencens origo samt dens fastgørelse
- Opstil legemerne
- Anvend hastighedsligningen
- Anvend evt. accelerationsligningen
- Sammenlign koefficienter

3.1 Generel bevægelse

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \overrightarrow{\omega^2} \cdot \vec{r}_{B/A}$$

$$\overrightarrow{V_p} = \vec{\omega} \times \vec{r}_{P/Ø}$$

3.2 Translation

$$\vec{v}_B = \vec{v}_A$$

$$\vec{a}_B = \vec{a}_A$$

4 Plan kinematik for et stift legeme (kraft og rotation)

- Tegn systemet op
- Indlæg koordinatsystem
- Tegn FLD og KD
- Newtons love
- Sammenlign ligninger og ubekendte
- Hvis der forekommer flere ubekendte end ligninger opstilles kinematiske ligninger for hvert legeme
- Ligningerne løses

4.1 Rotation om fast akse

$$\vec{V}_p = \vec{\omega} \times \vec{r}_{P/O} \qquad |\vec{v}_P| = |\vec{\omega}| \cdot |\vec{r}_P| \cdot \sin(\theta)$$

$$\vec{a}_P = \vec{\alpha} \times \vec{r}_{P/O} - \omega^2 \cdot \vec{r}_{P/O}$$

$$a_t = \alpha \cdot r \qquad \vec{a_t} = \vec{\alpha} \times \vec{r} \qquad v = \omega \cdot r$$

$$a_n = \omega^2 \cdot r \qquad \vec{a}_n = -\omega^2 \cdot \vec{r}$$

$$a_P = \sqrt{a_t{}^2 + a_n{}^2} \qquad \vec{a}_P = \vec{a_t} + \vec{a_n}$$

4.2 Roterende referencesystem

$$\vec{v}_B = \vec{v}_A + \vec{\Omega} \times \vec{r}_{B/A} + \vec{v}_{rel}$$

$$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{B/A} - \Omega^2 \cdot \vec{r}_{B/A} + 2\vec{\Omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

5 Kinetik (Newton)

5.1 Retlinet translation

x-y koordinatsystem

$$\sum \vec{F} = m \cdot \vec{\bar{a}}_c$$

$$\sum F = m \cdot \bar{a}_x$$

$$\sum F = m \cdot \bar{a}_y$$

$$\sum \vec{F} = m \cdot \vec{\bar{a}}_c$$

$$\sum M_G = 0$$

$$\sum M_P = \sum (M_K)_P$$

5.2 Kurvelineær translation

n-t koordinatsystem

$$\sum F_n = m \cdot \bar{a}_n$$

$$\sum F_t = m \cdot \bar{a}_t$$

$$\sum M_G = 0$$

$$a_n = \frac{v^2}{\rho}$$

5.3 Rotation om fast akse

n-t koordinatsystem

$$\sum F_n = m \cdot \bar{a}_n = m \cdot \omega^2 \cdot r_G$$

$$\sum F_t = m \cdot \bar{a}_t = m \cdot \alpha \cdot r_G$$

$$\sum M_G = I_G \cdot \alpha$$

$$\sum M_O = I_O \cdot \alpha$$

5.4 Generel bevægelse

x-y koordinatsystem

$$\sum F_x = m \cdot \bar{a}_x$$

$$\sum F_y = m \cdot \bar{a}_y$$

$$\sum M_G = I_G \cdot \alpha \qquad \sum M_P = \sum (M_K)_P$$

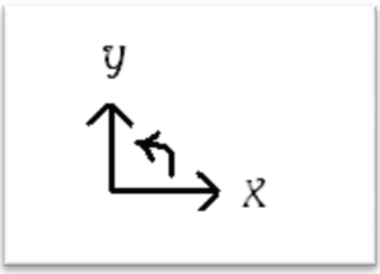
5.5 Steiners flyttesætning

$$I = I_G + md^2$$

d = vinkelret afstand mellem de to akser og der må KUN flyttes fra tyngdepunktet.

Mit sidste gode råd

Husk ALTID at vise hvilken retning du regner!



God fornøjelse☺!